

# Wheel flat detection using wayside measurements and Hilbert-Huang Transform

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**ABSTRACT:** Identifying wheel flats is critical in preventing train derailments and expensive repairs. This study utilized wayside monitoring and an uniaxial accelerometer, capable of  $\pm 500$  g peak at 10 kHz, attached to the foot of the rail mid-span between sleepers to capture the vibration pattern. Field measurements were conducted using trains with in-service generated flats and a train provided in as-new condition to establish a baseline response. Time-domain analyses were performed, indicating that the impulsive nature of a wheel flat striking the rail head leads to higher acceleration levels. To detect wheel flats, empirical mode decomposition and the Hilbert-Huang Transform (HHT) were applied to nonlinear and non-stationary filtered signals. Some of the train sets made available to the study had documented wheel flats, and in some cases, the wheel sets were subjectively and visually analysed, and notes were made of their condition. The presented method was able to identify and flag in the time-domain the bogie housing the defective wheel at the two different speeds investigated (10 and 30 km/h) for the reported wheel flats. Additional wheel flats were detected, however; without comprehensive reports detailing the condition of individual wheels, a conclusive result could not be established. The findings suggest that the HHT method can successfully identify the vibration signature of wheel flat impacts when measured wayside and justifies further research to unlock its potential for wheel flat detection.

## 1 INTRODUCTION

Smart railway monitoring is of increasing importance in a rapidly digitizing environment which seeks operational gains and maintenance advancements for commercial gain and increased sustainability. Wheel maintenance and replacement costs are a high expense for rolling stock operators and remain an area of active research within the umbrella of condition monitoring, estimating, and forecasting. Deviations in wheel profiles may be categorised as localised defects or defects that affect the full wheel circumference (Mosleh et al 2022b).

Wheel flats are discrete features on the wheel tread where it has been worn flat from a sliding incident. An impulsive noise is produced analogous to hammering on the rail head by a person stationed on the moving car. The strength of the impact and the frequency of repetition is proportional to the train speed and axle loading (Remington et al 1975; Jing 2018), both which aggravate ground-borne vibration and noise (Hanson et al 2006) further elevating the noise and vibration levels driving dissatisfaction in urban areas higher (Lakušić & Ahac 2012). With rail operations approaching the limit of wheel-rail traction in acceleration and deceleration driven by ever tighter

time schedules, the wheel flat issue of old remains a current research interest (Steenbergen 2008).

Wheel flat identification research is divided into two groups; wayside methods and on-board methods. On-board methods monitor the wheel status in real-time by placing the sensors on the vehicle components such as the axle box. The potential for real-time feedback after a sliding incident has occurred is an attractive proposition for early wheel flat detection and has garnered recent research interest (Bernal et al 2019; Liu et al 2022). The drawback of instrumenting the rolling stock fleet is however a consideration for the wayside monitoring method where discrete locations along the rail are instrumented. Naturally, these two methods yield overlap when the digital signal processing of the measured data is considered and as such research on either type could complement one another in certain areas.

The vibration characteristic of a wheel flat is nonlinear and non-stationary, making it difficult to analyse and identify the impulsive signature accurately. Non-stationary signals in particular do not lend themselves well to decomposition into sinusoidal components (Boashash 1992). As a result, much research has been dedicated to developing advanced signal

processing methods and stress-based sensor configurations for wheel flat identification. These methods aim to address the challenges posed by the nonlinear and non-stationary nature of the vibration signal, enabling more accurate detection and diagnosis of wheel flats (Alemi et al 2017; Jiang & Lin 2018; Bernal et al 2019; Fu et al 2023). The potential presence of an undetermined phase difference between flats on a single wheel set, or across multiple wheel sets with flats, further complicates the impulse detection search process.

Various methods have been used for successful wayside wheel flat detection; Fast Fourier transforms (FFT) and an array of ten sensors installed over a length covering the circumference of the wheel (Zhou et al 2020); Wavelet transforms have been used in various forms for detecting wheel flats, including a discrete transform to detect bogies containing wheel sets with wheel flats of varying severity using a single wayside accelerometer and a passenger train set (Belotti et al 2006), a wavelet transform and support vector machines to find a function that can predict the wheel flat using data from commercial wheel load checkpoints (Krummenacher et al 2018), a wavelet optimized for local extrema detection and a Wigner-Ville transform to identify wheel flats on operational trains using a single 3-axis accelerometer but with a limited dataset and unknown wheel conditions (Barman & Hazarika 2020); cepstrum analysis method on passenger trains where bogies with wheel flats were successfully detected using single sensor wayside vibration data during runs with a known and constant speed (Bracciali & Cascini 1997). Various other signal processing techniques have been applied successfully to simulated models; envelope spectrum analysis and an array of 12 wayside mounted strain gauges (Mosleh et al 2021); in a follow-up paper they replaced those strain gauges with two accelerometers and applied spectral kurtosis analysis (Mosleh et al 2021).

Empirical Mode Decomposition (EMD) is an adaptive signal analysis method without the requirement of a predetermined basis function (Zhao et al 2012). The Hilbert-Huang Transform (HHT) is based on the EMD method, which decomposes a signal into its intrinsic mode functions (IMFs). These IMFs can lead to mode mixing which implies that either a single IMF consists of signals with dramatically disparate scales or a signal of the same scale appears in different IMF components (Li et al 2016). The HHT applies the Hilbert transform to each IMF to obtain the instantaneous frequency, allowing it to analyse complex, nonlinear, and non-stationary signals. HHT has successfully been applied to simulated axle box vibrations (Li et al 2012; Jiang & Lin 2018; Souza et al 2022) including identifying two flats on a wheel and investigating the results of adding a phase difference between wheel flats (Jiang & Lin 2018).

Recently research has focused on methods incorporating machine learning for signal processing or feature classification. Convolutional neural networks were applied to time-series data from commercial wheel load checkpoints with success (Krummenacher et al 2018). A five-step process where unsupervised feature classification was used to determine that six accelerometers in series are required to identify wheel flats with a low-likelihood of misclassification (Mosleh et al 2022a). This work was built on by comparing the accuracy of four different feature extraction techniques using an unsupervised learning methodology to automatically detect a defective wheel and found that a single accelerometer on the rail is sufficient for identifying a defective wheel (Mohammadi et al 2023). A single accelerometer and an unsupervised early damage detection methodology, capable of automatically distinguishing a defective wheel from a healthy one (Mosleh et al 2022b).

The present work is carried out within the context of a rolling stock operator where current practice relies on detection of wheel flats through feedback from rail officials or the train driver. It is proposed to better this situation through the automated detection and localisation of early-stage wheel flats from wayside measurements.

The specific train model under investigation is designed to be resilient against the incidence of wheel flats owing to an electrically controlled pneumatic brake system which avoids wheel locking during brake operations. Suspected leaks in valves and incorrect handbrake release procedures have given rise to wheel flats forming

The bulk of previous research has relied on laboratory-based engineering test rigs with machined wheel flats, simulated data generated using dynamic numerical models, or focused on on-board axle box measurements. This current paper aims to explore the use of wayside acceleration detection as a means of identifying wheel flats in field-measured data that has been generated during the operation of a railway system. Specifically, the study seeks to identify and analyse defects that have been generated during normal usage of the railway system, using wayside acceleration measurements as a tool for data capturing and the Hilbert-Huang transform for analysis. The contribution of this paper is adapting the HHT, which has successfully been used on simulated axle box vibrations, to using full-scale field measurements of train sets containing multiple wheel flats across different wheel sets of varying severity and the ability to automatically detect wheel flats. One key approach of the present work on the measured data front is that high frequency measurements are utilized, precisely to hone in on reports that wheel flats cause high frequency impacts that damage rolling stock and rail infrastructure. This requires an investment in more sophisticated rail measurement sensors and equipment. The premise

constitutes an approach where high quality data is obtained to enable advanced signal processing and enhanced diagnostics. The proposed investigation analyses the acceleration profile associated with normal versus wheel flat measurements and distinguish normal wheels from damaged wheels.

## 2 HILBERT HUANG TRANSFORM

The requirement to characterize non-stationary signals for fault detection has resulted in the adoption of transient time-frequency analysis techniques (Huang et al 1998; Yan & Gao 2006; Barman & Hazarika 2020). The Hilbert-Huang Transform (HHT) is a continuous time-frequency analysis method which decomposes a signal into a set of intrinsic mode functions (IMFs) and a residue component called the residual, which captures any high-frequency noise or trend in the signal (Huang et al 1998). This work leverages the HHT technique which is receiving increasing attention for its ability to characterize non-linear, non-Gaussian, and non-stationary signals (Li et al 2012, Souza et al 2022).

In essence the Hilbert-Huang Transform follows a two-step process: Empirical Mode Decomposition to generate the Intrinsic Mode Functions and Hilbert Spectral Analysis to analyse the instantaneous frequency and energy density of each IMF (Huang et al 1998; Yan & Gao 2006; Souza et al 2022).

## 3 EXPERIMENTAL METHOD AND MEASUREMENTS

### 3.1 Track

Measurements were conducted on a straight and level section of track on Line 8 at the PRASA Paarden Eiland Rolling Stock Depot close to Cape Town, South Africa. The sleepers are of type P2 concrete sleepers with a spacing of 700 mm. The 48 kg/m rails are secured by Pandrol E-clips.

The test site was selected as it was a convenient section to gain access to several trains of interest for a pilot study. The implication of work within the depot is that the maximum speed of testing was constrained for safety reasons.

### 3.2 Rolling stock

The Xtrapolis Mega train set (TS) comprises two trailer cars (T-car, TC), one at each of the front and rear of the train and four motorized (M-car, M) passenger cars located between the T-cars. There are 16 motorized axles and 8 trailing axles with undercarriage measurements provided in Table 1. Apart from the operator and one team member the trains were empty for the duration of the measurement exercise,

resulting in a low axle loading. Infrastructure limitations on the equipment required to reprofile the wheels results in wheel diameters very close to as-new size.

Table 1. Undercarriage dimensions for Xtrapolis Mega

Distance between bogies on T-car	13,800 mm
Distance between bogies on M-car	14,600 mm
Distance between two axles	2,400 mm
Wheel diameter (New)	840 mm

Four Xtrapolis Mega TS with various wheel set conditions were made available to this study.

TS009 – Wheel flats are reported in TC1 on axles 1 and 2. The flats ranged in size between 25-35 mm which is less than the allowable maximum size of 40 mm. The train was released for service.

TS019 - The investigation report notes several wheel skid incidents owing to a brake fault on bogie 2 of TC1. The wheels on this bogie were repeatedly subjected to skidding and lead to extensive tread damage around its circumference, Figure 1. Maintenance protocols required this train set to be removed from service.



Figure 1. Wheel of TS019 showing extensive damage

TS101 - This new train set was not reported to have any flats.

TS104 - This new train set was not reported to have any flats. However, through subjective observations of the measurement team flats were heard and observed on M3 with softer sounds emitted from M2 and M4. As such this train does not suit its intended role as a benchmark for "as new" wheel condition.

### 3.3 Measuring equipment

Reports that wheel flats cause high frequency impacts that damage rolling stock and rail infrastructure require that sensors capable of high frequency, uniaxial measurements are utilized. A Model PCB 353B15 ICP,  $\pm 500$  g pk, accelerometer was placed on the foot of the rail, midspan between two sleepers capable of

delivering reliable measurements  $\pm 5\%$  for frequencies 0.5 to 10 kHz. Siemens LMS Test.Xpress software and LMS SCADAS data acquisition software was used and a sample rate of 51.2 kHz. Measurements were started when it was confirmed with the train driver that the test is ready and the environment is safe. Three constant speed test runs were repeated for each of the desired individual speeds.

## 4 WHEEL FLAT IDENTIFICATION

### 4.1 Subjective observation

The ground truth for wheel flats in the present study was based off maintenance reports and wheel flats characterised for that purpose. Subjective observations during the train pass-by tests pointed to further wheel flats being present. These additional defects were not individually characterised but were observed to be diminutive in size ( $< 20$  mm) during sporadic checks and their position on the train set noted. The train layout and markings of known and observed flats are given in Figure 2.

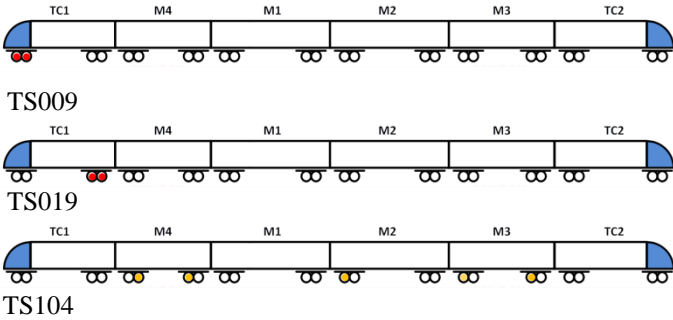


Figure 2. Observed and reported wheel flats on three test train sets. Red markers indicate flats that were reported as severe in maintenance reports, whereas less significant observed flats are indicated with yellow shading of the wheels in the diagram. TS101 had no observed or reported wheel flats.

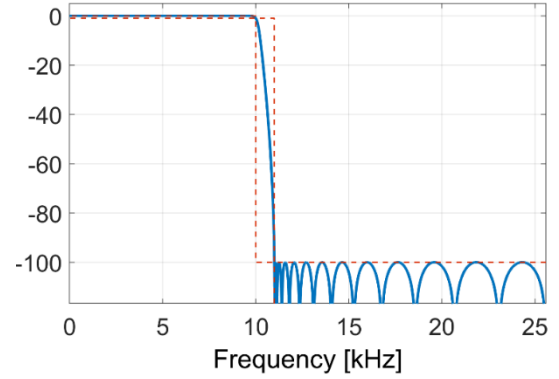
### 4.2 Time-domain analysis

For reference, conventional time-domain metrics are presented. Typical time-domain features include root-mean-square (r.m.s.), peak-to-peak, crest factor, kurtosis and skewness (Liang et al. 2013; Mosleh et al 2021; Mosleh et al 2022).

As a sample rate of 51.2 kHz was used in the present rig with a sensor capable of 10 kHz, frequency content beyond this range will be less accurate and should be filtered out. A Chebyshev Type II lowpass filter was used with a pass frequency of 10 kHz and stop frequency of 11 kHz. The stopband attenuation was set to 100 dB as presented in Figure 3. A 25<sup>th</sup> Chebyshev Type II filter was selected for its stability, maximally flat pass-band and relatively fast roll-off (Smith, 2013; Thompson, 2014). An overview of the measured results are presented in Table 2, time-domain metrics in **Error! Reference source not**

**found.**, and Figure 4 showing the measured signal relative to the train set.

The start time of measurements varied, which would affect r.m.s. metrics. The analysis was made more consistent by selecting measurement signals from the point where the moving r.m.s. (over 0.2 seconds, 0.1 second overlap) exceeded  $0.25 \text{ m/s}^2$  for speed cases 10, 20 and 30 km/h. For the 4 km/h tests a lower threshold of  $0.1 \text{ m/s}^2$  was required. These values were



determined by trial-and-error as potential triggering levels in future automated measurements.

Figure 3. Frequency response of the Chebyshev Type II signal conditioning filter for acceleration signals.

Table 2. Metrics from channel 1 for vertical acceleration measurements during train pass-by [ $\text{m/s}^2$ ]. Raw acceleration was filtered with a Chebyshev filter.

Speed [km/h]	r.m.s	Max	Min	STD	Skewness
Train set 009					
4	0.20	7.72	-9.66	0.20	-0.14
10	1.55	41.8	-94.7	1.55	-1.01
20	4.19	135.8	-270.5	4.19	-1.41
30	10.3	486.0	-796.2	10.3	-1.54
Train set 019					
4	0.33	12.2	-19.3	0.33	-0.52
10	2.05	61.9	-188.7	2.05	-1.77
20	8.00	287.2	-533.3	8.00	-1.86
30	14.6	482.3	-901.3	14.6	-0.74
Train set 101					
4	0.25	4.94	-5.22	0.25	-0.06
10	1.42	20.9	-31.8	1.42	-0.05
20	4.27	70.6	-108.7	4.27	-0.08
30	7.38	252.7	-240.9	7.38	-0.04
Train set 104					
4	0.23	11.0	-21.1	0.23	-1.68
10	1.41	54.4	-141.7	1.41	-2.86
20	4.11	150.9	-319.6	4.11	-1.96
30	9.09	293.7	-846.0	9.09	-2.05

## 5 HHT

The signal processing was performed on a single channel using MATLAB. The IMFs were generated using  $[\text{imf}, \text{residual}] = \text{emd}(x)$  and the HHT using  $[\text{hs}, \text{f}, \text{t}, \text{imfinsf}, \text{imfinse}] = \text{hht}(\_\_)$ . The instantaneous frequency and energy

Table 3. Time-signal graphs of the four train sets tested at various different speeds (The information in brackets indicates which TC entered the test region first)

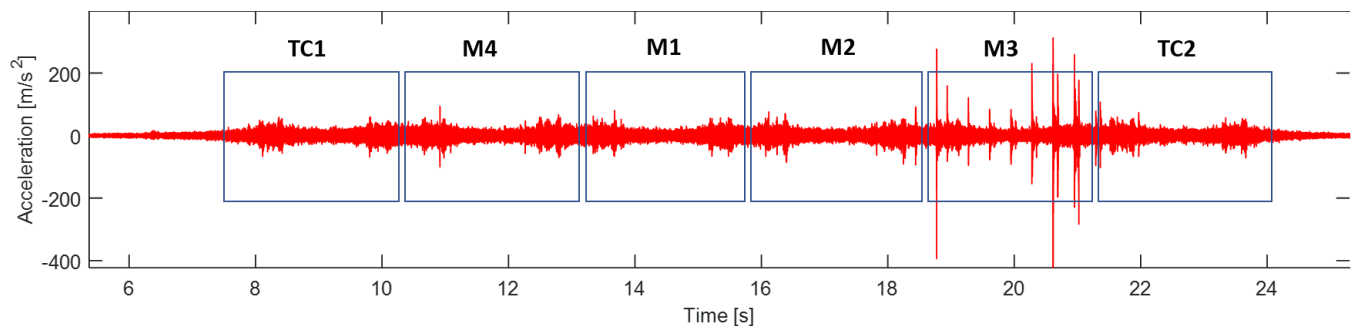
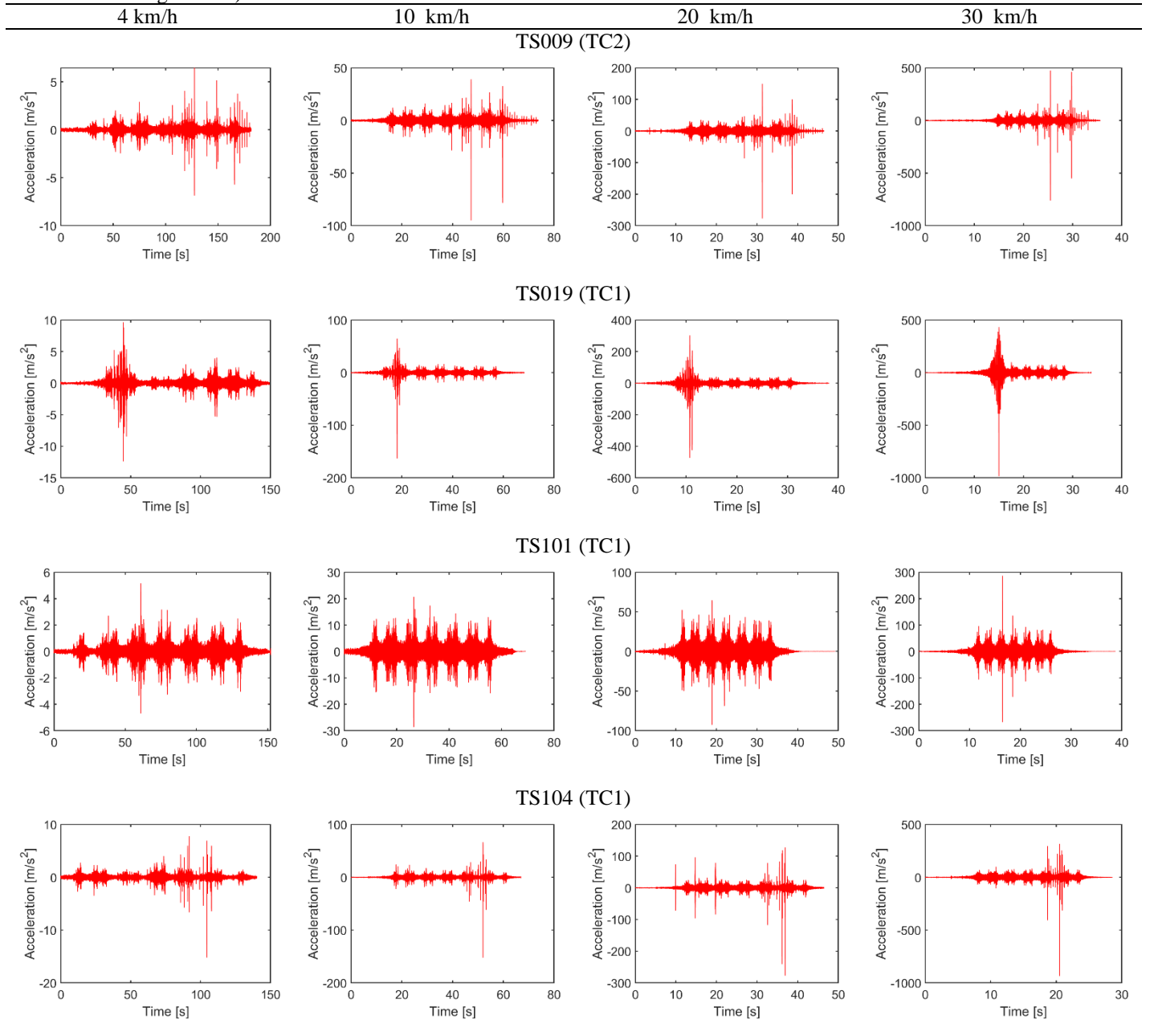
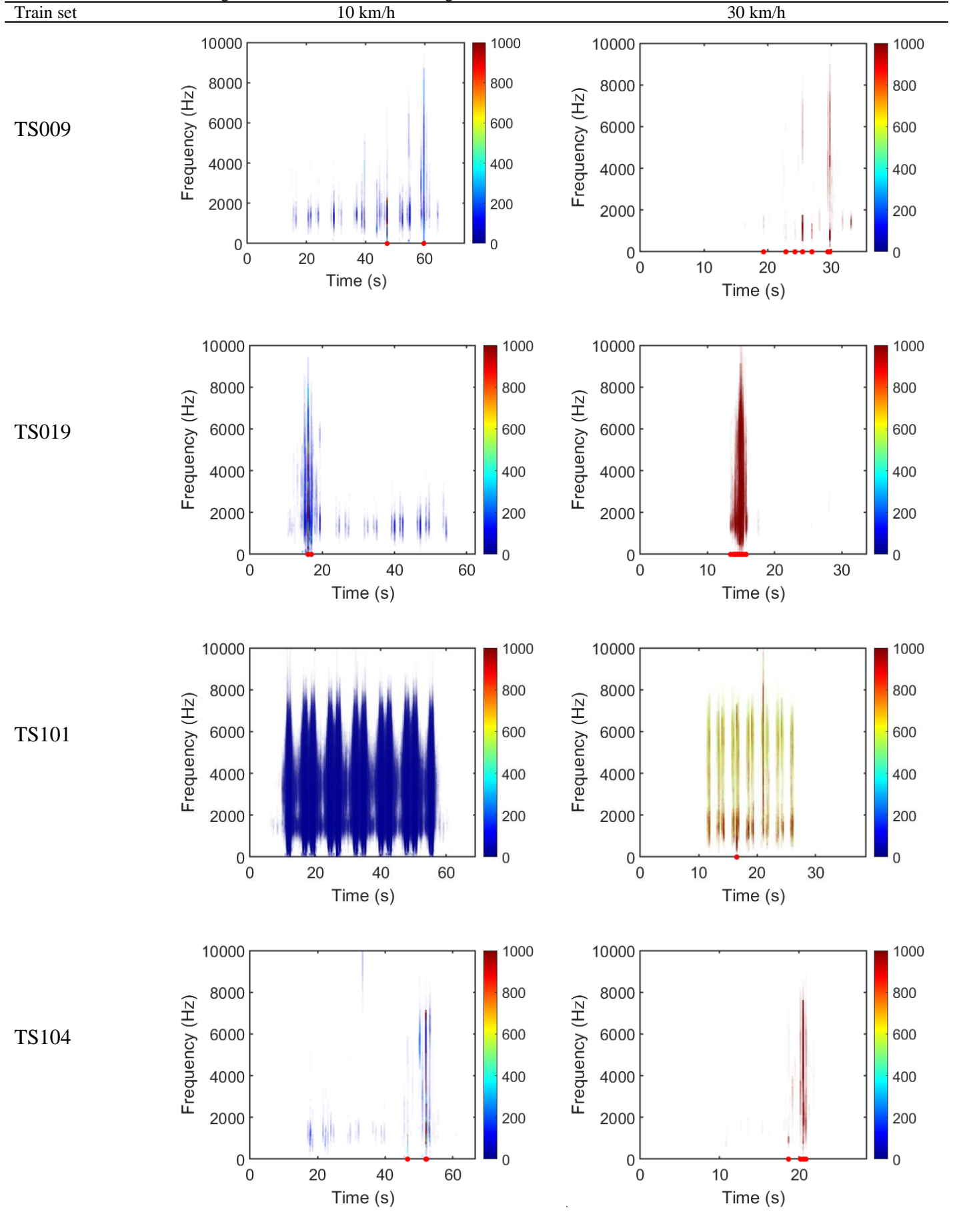


Figure 4. Relative location of the train set (TS104) shown in conjunction with the measured acceleration-time signal at 30 km/h high sampling rate used and will indicate each of these as a potential flat. The recorded time in the data set of each impulse identified as a wheel flat is output which can be used to determine the wheel sets at fault. The HHT of the four train sets at 10 km/h and 30 km/h are given in Table 4 along with the timestamps of identified wheel flats.

(imfinsf, imfinse) are used to detect wheel flats; an empirically determined threshold is set on both values before they are multiplied with each other resulting in a vector. A searching and grouping algorithm was created to search for impulses and group them together within an empirically determined time frame; a single flat will excite a range a data points due to the



Table 4. HSA of IMF including bullets on the x-axis indicating calculated wheel flat locations



## 6 DISCUSSION

The filtered acceleration-time signals in **Error! Reference source not found.** & **Error! Reference source not found.** show a clear distinction between the magnitudes of acceleration levels of the train sets with documented flats (TS009 & 019) versus the set with wheels in as-new condition (TS101). This is characteristic of the impulsive nature of the wheel flat making contact with the rail head. The relative location of the train carriages imposed on an acceleration-time signal in Figure 4 further illustrates the ability of the peaks to be measured and used to identify the passage of each bogie. The smoother running wheels of TS 101 allow for individual wheel sets to be identified. The necessity for using sensors with the high-rated capabilities used for these measurements are clear at 30 km/h with the acceleration approaching 100 g.

The HHT method showed its ability at detecting high energy – high frequency occurrences and flag them as potential wheel flats in the time-domain. The thresholds used on the IMFs frequency and energy content need to be adapted according to the speed and follows an empirical process in determining their values. Incorrect selection here will result in wheel flats either not making the criteria or identifying false flats. A summary of the four train sets tested:

TS009: The bogie with wheel flats in the investigation report was identified. Further incidences were identified in the 30 km/h data between 20-30 s. The train speed is not known accurately enough to back-calculate the time between those repeated flags to the distance between wheels or bogies indicating to a single flat being measured repeatedly. Both the 10- and 30 km/h data also identified a flat on M4 which corresponds to subjective accounts while capturing the data.

TS019: The bogie with wheel flats in the investigation report was identified. These flat were classified as severe and lead to a high energy impulse. Flats were flagged in short succession at both the speeds investigated. The time delay between flagged events did not clearly correspond to full wheel rotations. It was noted from the investigation report that these wheel sets were subjected to severe flats along the circumference of the wheel. It is therefore possible that multiple flats were detected during every rotation of the wheel and that high-frequency content was smeared.

TS101: Subjectively this train set did not have any audible flats and was presented in an as-new wheel condition. The HHT shows the passage of each bogie and wheel set clearly at similar energy content. A flat was flagged in the 30 km/h data on M2, axle 1 or 2. The condition of these wheel sets could not be confirmed,

TS104: Multiple occurrences were flagged at both the investigated speeds. These correspond with visual inspections on M4 which had flats < 20 mm.

The ability of the HHT to detect wheel flats and flag them in the time-domain is evident at the two different speeds researched. Without utilising the strength of this method which includes the ability to investigate individual IMFs its potential cannot completely be unlocked.

The on-board governing and indicating system of the train was used to classify the speed. Given the section of rail available and inevitable variations between subsequent test runs it is not certain that the target speed was met and maintained. The data should be processed such that passing wheels and the various geometric quantities of the bogies are used to calculate a more accurate speed with the data that is available. Doing this will allow for repeat flats that are measured to be filtered out from the reported defective wheel set list.

## 7 CONCLUSION

This study presents an approach for detecting wheel flats on trains, utilizing high-frequency and acceleration level accelerometers and the Hilbert-Huang Transform. Our findings demonstrate the successful identification of known reported flats as well as flats which were subjectively identified next to the passing train. The use of actual wayside measurements in this study provides a more accurate and realistic assessment of wheel condition, taking into account the complex dynamics of real-world train operations and the effect of having multiple flats on a single wheel, as well as multiple flats across multiple bogies. Studies have shown that the HHT method is capable of identifying wheel flats when on-board measurements are used but have not been applied to wayside conditions.

However, our study highlights the challenges in accurately detecting and confirming the actual number of flats due to limitations in obtaining true information on the actual wheel condition. Despite this limitation, our approach offers valuable insights into the detection and monitoring of wheel flats, which can improve train safety and maintenance procedures.

Further research is needed to address the limitations of this study:

1. Inspect the condition of each wheel on every train set to improve the ground truth and improve accuracy of identifying wheel flats.
2. Use individual IMFs at detecting wheel flats instead of their total representation.
3. Calculate speed using the passing of each wheel.
4. Set the thresholds to identify small flats, but also be able to then filter out detecting repeats of more severe flats.

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